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One-loop renormalization of general $\mathcal{N} = \frac{1}{2}$ supersymmetric gauge theory

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We investigate the one-loop renormalizability of a general $\mathcal{N} = \frac{1}{2}$ supersymmetric gauge theory coupled to chiral matter, and show the existence of an $\mathcal{N} = \frac{1}{2}$ supersymmetric $SU(N) \otimes U(1)$ theory which is renormalizable at one loop.

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I. INTRODUCTION

There has recently been much interest in theories defined on nonanticommutative superspace [1–4]. Such theories are nonhermitian and turn out to have only half the supersymmetry of the corresponding ordinary supersymmetric theory—hence the term "$\mathcal{N} = \frac{1}{2}$ supersymmetry". These theories are not power-counting renormalizable[1] but it has been argued [7–10] that they are in fact nevertheless renormalizable, in the sense that only a finite number of additional terms need to be added to the lagrangian to absorb divergences to all orders. This is primarily because although the theory contains operators of dimension five and higher, they are not accompanied by their hermitian conjugates which would be required to generate divergent diagrams. This argument does not of course guarantee that the precise form of the lagrangian will be preserved by renormalization; nor does the $\mathcal{N} = \frac{1}{2}$ supersymmetry, since some terms in the lagrangian are inert under this symmetry. Moreover, the argument also requires (in the gauged case) the assumption of gauge invariance to rule out some classes of divergent structure. As we showed in Ref. [11], there are problems with this assumption; even at one loop, at least in the standard class of gauges, divergent non-gauge-invariant terms are generated. However, in the case of pure $\mathcal{N} = \frac{1}{2}$ supersymmetry (i.e. no chiral matter) we displayed a divergent field redefinition which miraculously removed the non-gauge-invariant terms and restored gauge invariance. Moreover, we displayed a slightly modified (but still $\mathcal{N} = \frac{1}{2}$ supersymmetric) version of the original pure $\mathcal{N} = \frac{1}{2}$ lagrangian which had a form preserved under renormalization. The authors of Ref. [12] obtained the one-loop effective action for pure $\mathcal{N} = \frac{1}{2}$ supersymmetry using a superfield formalism. Although they found divergent contributions which broke supergauge invariance, their final result was gauge-invariant without the need for any redefinition. On the other hand it is hard to make any inferences about renormalizability from their superfield form of the one-loop result. In the present work we consider the $\mathcal{N} = \frac{1}{2}$ supersymmetric action coupled to chiral matter. The original nonanticommutative theory defined in superfields appears to require a $U(N)$ gauge group [4,6]. In Ref. [11] we considered the component form of the pure $\mathcal{N} = \frac{1}{2}$ supersymmetric action adapted to $SU(N)$. We argued that it was only for $SU(N)$ that a form-invariant lagrangian could be defined; indeed the $U(N)$ gauge symmetry is not preserved under renormalization. In the case with chiral matter it turns out that the lagrangian is no longer form-invariant in the $SU(N)$ case either. In fact, a general $\mathcal{N} = \frac{1}{2}$ supersymmetric $SU(N)$ invariant action cannot be defined. However, we shall demonstrate the existence of a new $\mathcal{N} = \frac{1}{2}$ supersymmetric $SU(N) \otimes U(1)$ action which is renormalizable and preserves $\mathcal{N} = \frac{1}{2}$ supersymmetry at one loop.

The action for an $\mathcal{N} = \frac{1}{2}$ supersymmetric $U(N)$ gauge theory coupled to chiral matter is given by [4]

$$\begin{aligned} S = \int d^4x\bigg[&\mathrm{tr}\left\{-\frac{1}{2}F^{\mu\nu}F_{\mu\nu} - 2i\bar{\lambda}\bar{\sigma}^{\mu}(D_{\mu}\lambda) + D^2\right\} - 2igC^{\mu\nu}\mathrm{tr}\,\{F_{\mu\nu}\bar{\lambda}\,\bar{\lambda}\} + g^2|C|^2\mathrm{tr}\,\{(\bar{\lambda}\,\bar{\lambda})^2\} \\ &+ \bigg\{\bar{F}F - i\bar{\psi}\bar{\sigma}^{\mu}D_{\mu}\psi - D^{\mu}\bar{\phi}D_{\mu}\phi + g\bar{\phi}D\phi + i\sqrt{2}g(\bar{\phi}\lambda\psi - \bar{\psi}\,\bar{\lambda}\,\phi) + \sqrt{2}gC^{\mu\nu}D_{\mu}\bar{\phi}\,\bar{\lambda}\,\bar{\sigma}_{\nu}\psi \\ &+ igC^{\mu\nu}\bar{\phi}F_{\mu\nu}F + \frac{1}{4}|C|^2g^2\bar{\phi}\,\bar{\lambda}\,\bar{\lambda}\,F + (\phi \to \tilde{\phi}, \psi \to \tilde{\psi}, F \to \tilde{F}, R^A \to -(R^A)^*, C^{\mu\nu} \to -C^{\mu\nu})\bigg\}\bigg], \end{aligned} \tag{1.1}$$

where we include a multiplet $\{\phi, \psi, F\}$ transforming according to the fundamental representation of $U(N)$ with group matrices R^A and, to ensure anomaly cancellation, a multiplet $\{\tilde{\phi}, \tilde{\psi}, \tilde{F}\}$ transforming according to its conjugate. The change $C^{\mu\nu} \to -C^{\mu\nu}$ for the conjugate representation is due to the fact that the anticommutation relations for the conjugate fundamental representation differ by a sign from those for the fundamental representation. We define

[1] See Refs. [5,6] for other discussions of the ultraviolet properties of these theories.

$$D_\mu \phi = \partial_\mu \phi + igA_\mu \phi, \qquad D_\mu \lambda = \partial_\mu \lambda + ig[A_\mu, \lambda],$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu], \tag{1.2}$$

(with a similar expression for $D_\mu \tilde{\phi}$) where

$$A_\mu = A^A_\mu R^A, \qquad \lambda = \lambda^A R^A, \qquad D = D^A R^A. \tag{1.3}$$

The group matrices satisfy

$$[R^A, R^B] = if^{ABC} R^C, \qquad \{R^A, R^B\} = d^{ABC} R^C, \tag{1.4}$$

where d^{ABC} is totally symmetric. If one decomposes $U(N)$ as $SU(N) \otimes U(1)$ then our convention is that R^a are the $SU(N)$ generators and R^0 the $U(1)$ generator. Of course then $f^{ABC} = 0$ unless all indices are $SU(N)$. The matrices are normalised so that Tr $[R^A R^B] = \frac{1}{2}\delta^{AB}$. In particular, $R^0 = \sqrt{\frac{1}{2N}}1$. We note that $d^{ab0} = \sqrt{\frac{2}{N}}\delta^{ab}$, $d^{000} = \sqrt{\frac{2}{N}}$. In Eq. (1.1), $C^{\mu\nu}$ is related to the nonanticommutativity parameter $C^{\alpha\beta}$ by

$$C^{\mu\nu} = C^{\alpha\beta}\epsilon_{\beta\gamma}\sigma^{\mu\nu\gamma}_{\alpha}, \tag{1.5}$$

where

$$\sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu), \; \bar{\sigma}^{\mu\nu} = \frac{1}{4}(\bar{\sigma}^\mu \sigma^\nu - \bar{\sigma}^\nu \sigma^\mu), \tag{1.6}$$

and

$$|C|^2 = C^{\mu\nu} C_{\mu\nu}. \tag{1.7}$$

Our conventions are in accord with [3]; in particular,

$$\sigma^\mu \bar{\sigma}^\nu = -\eta^{\mu\nu} + 2\sigma^{\mu\nu}. \tag{1.8}$$

Properties of C which follow from Eq. (1.5) are

$$C^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\gamma}(\sigma^{\mu\nu})_\gamma{}^\beta C_{\mu\nu}, \tag{1.9a}$$

$$C^{\mu\nu}\sigma_{\nu\alpha\dot{\beta}} = C_\alpha{}^\gamma \sigma^\mu_{\gamma\dot{\beta}}, \tag{1.9b}$$

$$C^{\mu\nu}\bar{\sigma}_\nu^{\dot{\alpha}\beta} = -C^\beta{}_\gamma \bar{\sigma}^{\mu\dot{\alpha}\gamma}. \tag{1.9c}$$

Upon substituting Eq. (1.3) into Eq. (1.1) and using Eq. (1.4), we obtain the action in the $U(N)$ case in the form:

$$\begin{aligned} S = \int d^4x \Big[&-\frac{1}{4}F^{\mu\nu A}F^A_{\mu\nu} - i\bar{\lambda}^A\bar{\sigma}^\mu(D_\mu\lambda)^A + \frac{1}{2}D^AD^A - \frac{1}{2}igC^{\mu\nu}d^{ABC}F^A_{\mu\nu}\bar{\lambda}^B\bar{\lambda}^C + \frac{1}{8}g^2|C|^2 d^{ABE}d^{CDE}(\bar{\lambda}^A\bar{\lambda}^B)(\bar{\lambda}^C\bar{\lambda}^D) \\ &+ \Big\{ \bar{F}F - i\bar{\psi}\bar{\sigma}^\mu D_\mu\psi - D^\mu\bar{\phi}D_\mu\phi + g\bar{\phi}D\phi + i\sqrt{2}g(\bar{\phi}\lambda\psi - \bar{\psi}\,\bar{\lambda}\,\phi) + \sqrt{2}gC^{\mu\nu}D_\mu\bar{\phi}\,\bar{\lambda}\,\bar{\sigma}_\nu\psi \\ &+ igC^{\mu\nu}\bar{\phi}F_{\mu\nu}F + \frac{1}{8}|C|^2g^2d^{ABC}\bar{\phi}R^A\bar{\lambda}^B\bar{\lambda}^C F + (\phi \to \tilde{\phi}, \psi \to \tilde{\psi}, F \to \tilde{F}, R^A \to -(R^A)^*, C^{\mu\nu} \to -C^{\mu\nu})\Big\}\Big]. \end{aligned} \tag{1.10}$$

with gauge coupling g, gauge field A_μ, gaugino λ and with

$$F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - gf^{ABC}A^B_\mu A^C_\nu, \qquad D_\mu\lambda^A = \partial_\mu\lambda^A - gf^{ABC}A^B_\mu\lambda^C. \tag{1.11}$$

However, it is clear that the $U(N)$ action cannot be renormalizable, since for any $U(N)$ gauge theory the gauge couplings for the $SU(N)$ and $U(1)$ parts of the theory renormalize differently. To obtain a renormalizable theory one must introduce different couplings for the $SU(N)$ and $U(1)$ parts of the gauge group and then the $U(N)$ gauge-invariance is lost. This is a trivial point but one which does not seem to have been made in other discussions of the renormalization of $\mathcal{N} = \frac{1}{2}$ supersymmetric gauge theory. Remarkably, we shall see that by a judicious introduction of different couplings for the $SU(N)$ and $U(1)$ parts of the gauge group, we can obtain an $SU(N) \otimes U(1)$ theory which still has $\mathcal{N} = \frac{1}{2}$ supersymmetry which is preserved under renormalization. We propose replacing Eq. (1.10) by

$$\begin{aligned} S = \int d^4x \Big[&-\frac{1}{4}F^{\mu\nu A}F^A_{\mu\nu} - i\bar{\lambda}^A\bar{\sigma}^\mu(D_\mu\lambda)^A + \frac{1}{2}D^AD^A - \frac{1}{2}iC^{\mu\nu}d^{ABC}e^{ABC}F^A_{\mu\nu}\bar{\lambda}^B\bar{\lambda}^C + \frac{1}{8}g^2|C|^2 d^{abe}d^{cde}(\bar{\lambda}^a\bar{\lambda}^b)(\bar{\lambda}^c\bar{\lambda}^d) \\ &+ \frac{1}{4N}\frac{g^4}{g_0^2}|C|^2(\bar{\lambda}^a\bar{\lambda}^a)(\bar{\lambda}^b\bar{\lambda}^b) + \Big\{ \bar{F}F - i\bar{\psi}\bar{\sigma}^\mu D_\mu\psi - D^\mu\bar{\phi}D_\mu\phi + \bar{\phi}\,\hat{D}\,\phi + i\sqrt{2}(\bar{\phi}\,\hat{\lambda}\,\psi - \bar{\psi}\,\hat{\bar{\lambda}}\,\phi) \\ &+ \sqrt{2}C^{\mu\nu}D_\mu\bar{\phi}\,\hat{\bar{\lambda}}\,\bar{\sigma}_\nu\psi + iC^{\mu\nu}\bar{\phi}\hat{F}_{\mu\nu}F + \frac{1}{8}|C|^2d^{ABC}\bar{\phi}R^A\hat{\bar{\lambda}}^B\hat{\bar{\lambda}}^C F + \frac{1}{N}\gamma_1 g_0^2|C|^2(\bar{\lambda}^a\bar{\lambda}^a)(\bar{\lambda}^0\bar{\lambda}^0) \\ &- \gamma_2 C^{\mu\nu}g(\sqrt{2}D_\mu\bar{\phi}\bar{\lambda}^aR^a\bar{\sigma}_\nu\psi + \sqrt{2}\bar{\phi}\bar{\lambda}^aR^a\bar{\sigma}_\nu D_\mu\psi + i\bar{\phi}F^a_{\mu\nu}R^aF) \\ &+ (\phi \to \tilde{\phi}, \psi \to \tilde{\psi}, F \to \tilde{F}, R^A \to -(R^A)^*, C^{\mu\nu} \to -C^{\mu\nu})\Big\}\Big], \end{aligned} \tag{1.12}$$

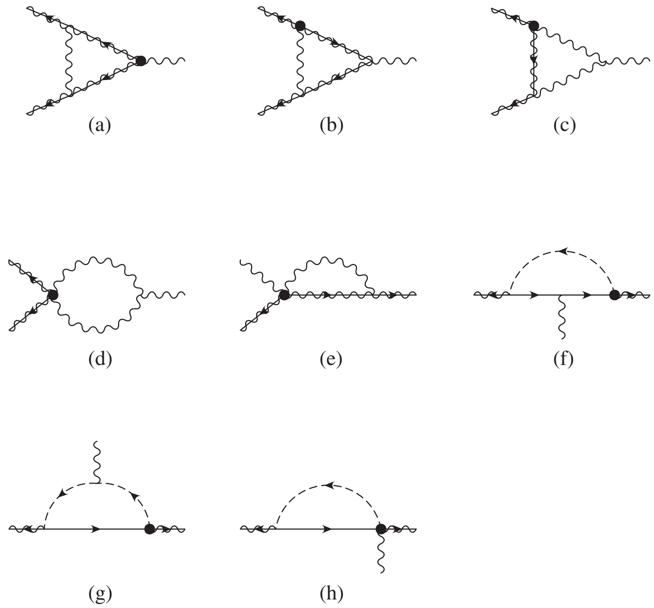

FIG. 1. Diagrams with one gauge, two gaugino lines; the dot represents the position of a C.

where γ_1, γ_2 are constants,

$$\hat{A}_\mu = \hat{A}^A_\mu R^A = g A^a_\mu R^a + g_0 A^0_\mu R^0, \tag{1.13}$$

with similar definitions for $\hat{\lambda}$, $\hat{D}$, $\hat{F}_{\mu\nu}$, and now

$$D_\mu \phi = (\partial_\mu + i\hat{A}_\mu)\phi. \tag{1.14}$$

We also have

$$e^{abc} = g, \quad e^{a0b} = e^{ab0} = e^{000} = g_0, \quad e^{0ab} = \frac{g^2}{g_0}. \tag{1.15}$$

It is easy to show that Eq. (1.12) is invariant under

$$\begin{aligned} &\delta A^A_\mu = -i\bar{\lambda}^A \bar{\sigma}_\mu \epsilon \\ &\delta \lambda^A_\alpha = i\epsilon_\alpha D^A + (\sigma^{\mu\nu}\epsilon)_\alpha [F^A_{\mu\nu} + \frac{1}{2} i C_{\mu\nu} e^{ABC} d^{ABC} \bar{\lambda}^B \bar{\lambda}^C], \\ &\delta \bar{\lambda}^A_{\dot{\alpha}} = 0, \qquad \delta D^A = -\epsilon \sigma^\mu D_\mu \bar{\lambda}^A, \qquad \delta\phi = \sqrt{2}\epsilon\psi, \\ &\delta\bar{\phi} = 0, \, \delta\psi^\alpha = \sqrt{2}\epsilon^\alpha F, \\ &\delta\bar{\psi}_{\dot{\alpha}} = -i\sqrt{2}(D_\mu \bar{\phi})(\epsilon\sigma^\mu)_{\dot{\alpha}}, \qquad \delta F = 0, \\ &\delta\bar{F} = -i\sqrt{2} D_\mu \bar{\psi} \bar{\sigma}^\mu \epsilon - 2i\bar{\phi}\epsilon\hat{\lambda} + 2C^{\mu\nu} D_\mu(\bar{\phi}\epsilon\sigma_\nu \bar{\hat{\lambda}}). \end{aligned} \tag{1.16}$$

Apart from the term with the coefficient γ_1 and the group of terms with coefficient γ_2, Eq. (1.12) reduces to the original $U(N)$ lagrangian Eq. (1.10) derived from nonanticommuting superspace upon setting $g_0 = g$. These remaining terms are separately invariant under $\mathcal{N} = \frac{1}{2}$ supersymmetry and must be included to obtain a renormalizable lagrangian, as we shall see.

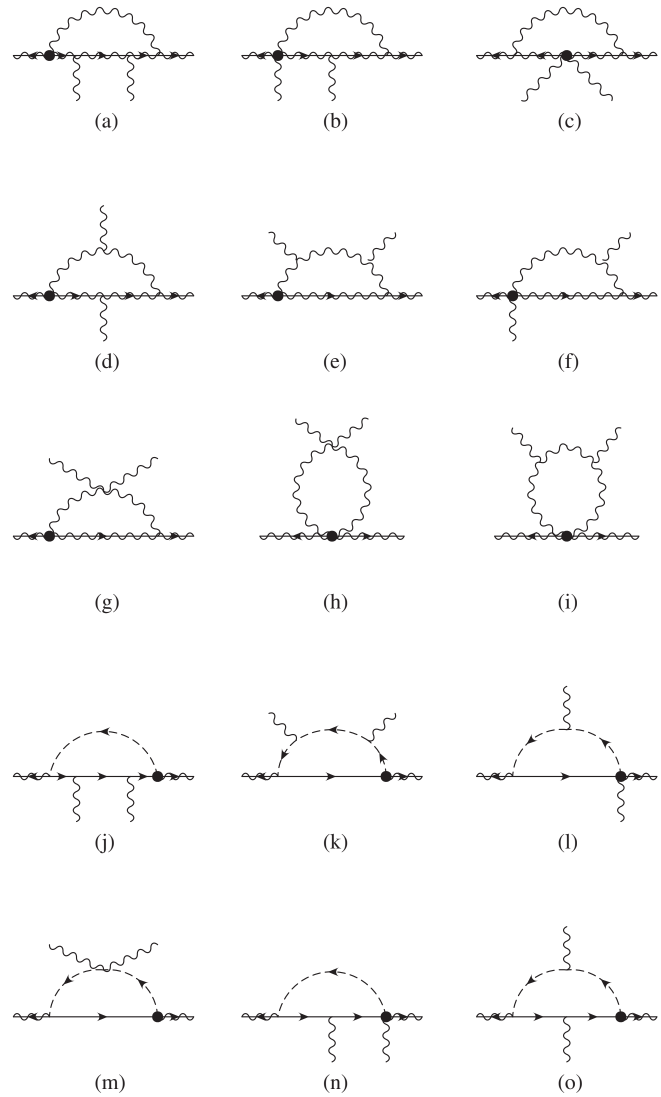

FIG. 2. Diagrams with two gauge and two gaugino lines; the dot represents the position of a C.

In Ref. [11] we gave an $SU(N)$-invariant theory with $\mathcal{N} = \frac{1}{2}$ supersymmetry in the pure gauge case. The supersymmetry transformations in that case were essentially

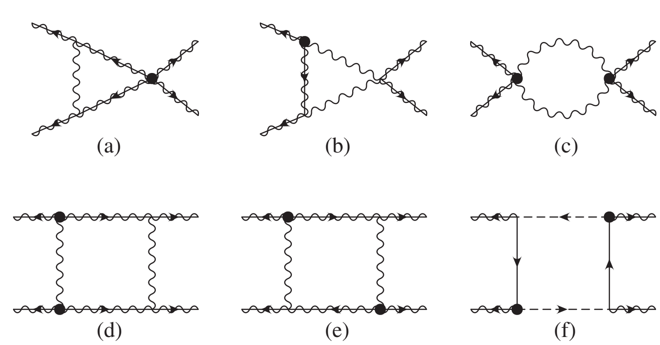

FIG. 3. Diagrams with four gaugino lines; the dot represents the position of a C *or* $|C|^2$.

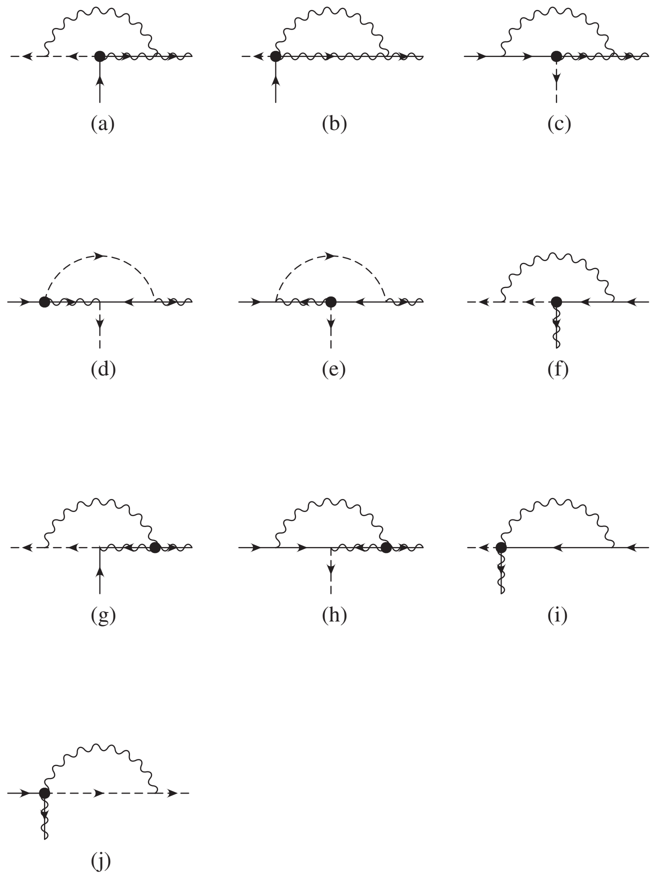

FIG. 4. Diagrams with one gaugino, one scalar and one chiral fermion line; the dot represents the position of a C.

obtained by striking out any 0 index in the $U(N)$ transformations. However in the general case these transformations do not close, since the gauge-field part of the $\sqrt{2}gC^{\mu\nu}D_\mu\bar{\phi}\,\bar{\lambda}\,\bar{\sigma}_\nu\psi$ term produces a $C^{\mu\nu}\bar{\phi}\bar{\lambda}^a\bar{\lambda}^a\bar{\sigma}_{\mu\nu}\psi$ term which in the $U(N)$ case is cancelled by the variation of $\bar{\phi}\lambda^0\psi$, a term which is absent for $SU(N)$. Of course because of the $\frac{g^2}{g_0}$ terms, one cannot obtain the $SU(N)$ theory simply by setting $g_0 = 0$ in the $SU(N)\otimes U(1)$ theory.

We use the standard gauge-fixing term

$$S_{\rm gf} = \frac{1}{2\alpha}\int d^4x(\partial.A)^2 \tag{1.17}$$

with its associated ghost terms. The gauge propagators for $SU(N)$ and $U(1)$ are both given by

$$\Delta_{\mu\nu} = -\frac{1}{p^2}\left(\eta_{\mu\nu} + (\alpha - 1)\frac{p_\mu p_\nu}{p^2}\right) \tag{1.18}$$

(omitting group factors) and the fermion propagator is

$$\Delta_{\alpha\dot{\alpha}} = \frac{p_\mu\sigma^\mu_{\alpha\dot{\alpha}}}{p^2}, \tag{1.19}$$

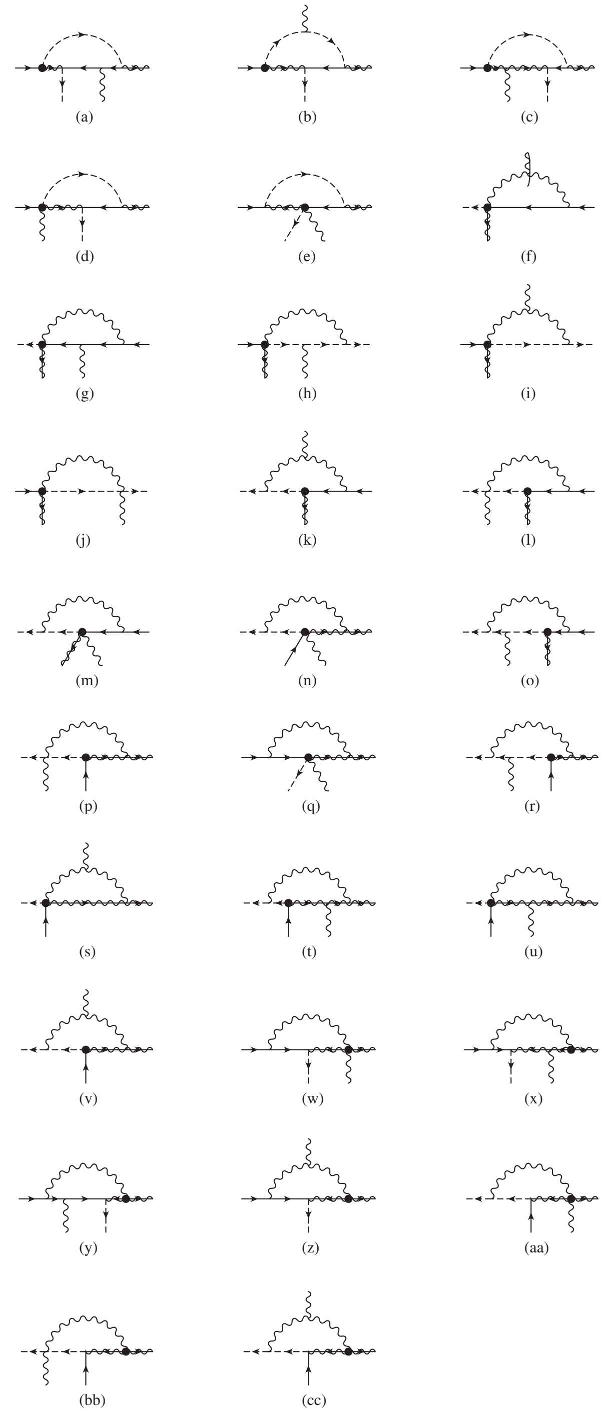

FIG. 5. Diagrams with one gaugino, one scalar, one chiral fermion and one gauge line; the dot represents the position of a C.

where the momentum enters at the end of the propagator with the undotted index. The one-loop graphs contributing to the "standard" terms in the lagrangian (those without a $C^{\mu\nu}$) are the same as in the ordinary $\mathcal{N} = 1$ case, so anomalous dimensions and gauge β-functions are as for $\mathcal{N} = 1$. Since our gauge-fixing term in Eq. (1.17) does not

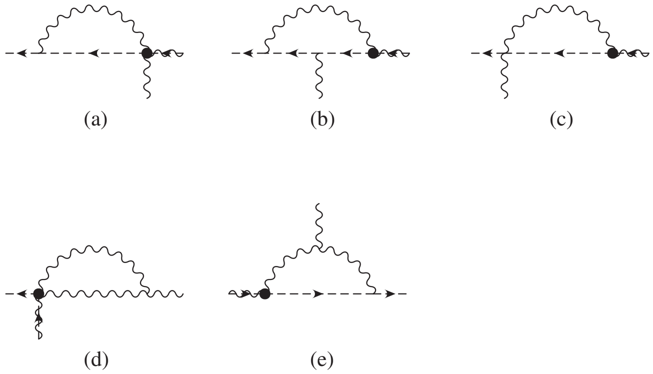

FIG. 6. Diagrams with one gauge, one scalar and one auxiliary line; the dot represents the position of a C.

preserve supersymmetry, the anomalous dimensions for A_μ and λ are different (and moreover gauge-parameter dependent), as are those for ϕ and ψ. However, the gauge β-functions are of course gauge-independent. The one-loop one-particle-irreducible (1PI) graphs contributing to the new terms (those containing C) are depicted in Figs. 1–8.

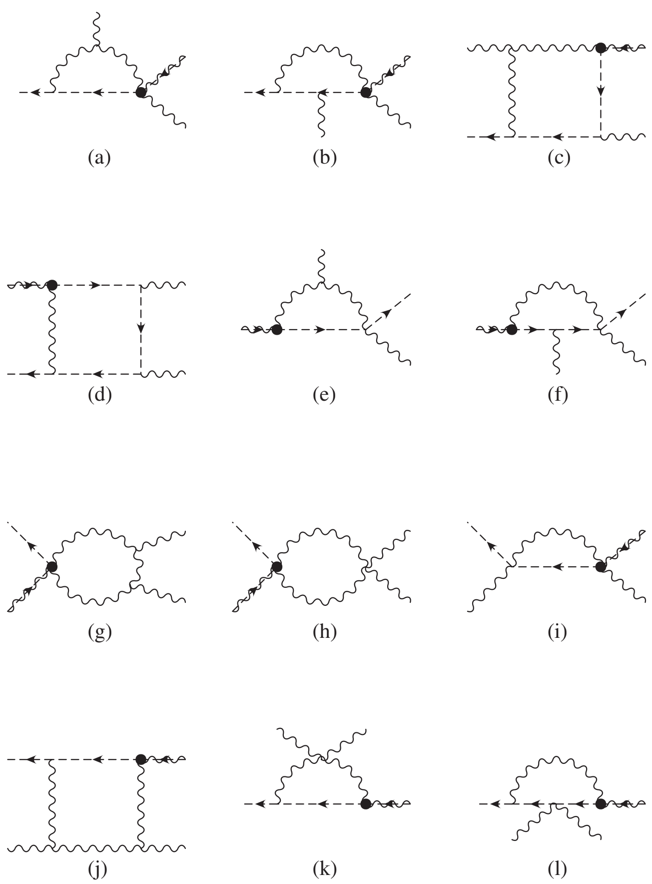

FIG. 7. Diagrams with two gauge, one scalar and one auxiliary line; the dot represents the position of a C.

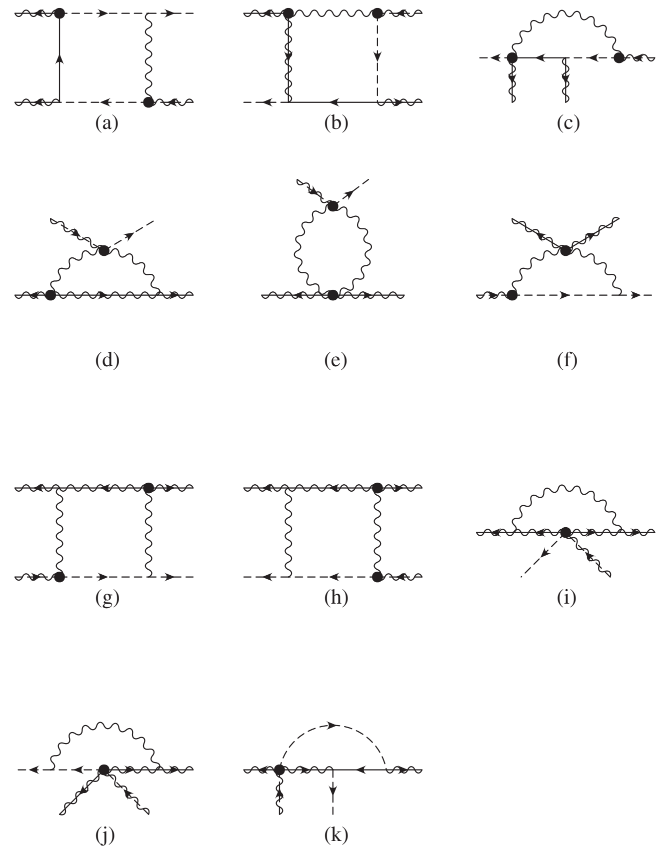

FIG. 8. Diagrams with two gaugino, one scalar and one auxiliary line; the dot represents the position of a C or a $|C|^2$.

II. RENORMALIZATION OF THE $SU(N) \otimes U(1)$ ACTION

Ordinarily the divergences in one-loop diagrams should be cancelled by the one-loop divergences in S_B, obtained by replacing the fields and couplings in Eq. (1.12) with bare fields and couplings given by

$$\begin{aligned}
&\lambda_B^a = Z_\lambda^{\frac{1}{2}} \lambda^a, \qquad \lambda_B^0 = Z_{\lambda^0}^{\frac{1}{2}} \lambda^0, \qquad A_{\mu B}^a = Z_A^{\frac{1}{2}} A_\mu^a, \\
&A_{\mu B}^0 = Z_{A^0}^{1/2} A_\mu^0, \qquad \phi_B = Z_\phi^{1/2} \phi, \qquad \psi_B = Z_\psi^{1/2} \psi, \\
&g_B = Z_g g, \qquad g_{0B} = Z_{g_0} g_0, \qquad \gamma_{1B} = Z_1, \\
&\gamma_{2B} = Z_2, \qquad C_B^{\mu\nu} = Z_C C^{\mu\nu}, \qquad |C|_B^2 = Z_{|C|^2} |C|^2.
\end{aligned} \tag{2.1}$$

In Eq. (2.1), Z_1 and Z_2 are divergent contributions, in other words we have set the renormalized couplings γ_1 and γ_2 to zero for simplicity. The other renormalization constants start with tree-level values of 1. As we mentioned before, the renormalization constants for the fields and for the gauge couplings g, g_0 are the same as in the ordinary $\mathcal{N} = 1$ supersymmetric theory and are therefore given up to one loop by [13]:

$$
\begin{aligned}
Z_\lambda &= 1 - g^2 L(2\alpha N + 2),\\
Z_A &= 1 + g^2 L[(3-\alpha)N - 2],\\
Z_g &= 1 + g^2 L(1 - 3N), \qquad Z_\phi = 1 + 2(1-\alpha)L\hat{C}_2,\\
Z_\psi &= 1 - 2(1+\alpha)L\hat{C}_2,
\end{aligned}
\tag{2.2}
$$

where (using dimensional regularization with $d = 4 - \epsilon$) $L = \frac{1}{16\pi^2\epsilon}$ and

$$\hat{C}_2 = g^2 R^a R^a + g_0^2 R^0 R^0 = \frac{1}{2}\left(Ng^2 + \frac{1}{N}\Delta\right) \tag{2.3}$$

with

$$\Delta = g_0^2 - g^2. \tag{2.4}$$

(For the gauge multiplet, we have given here the renormalization constants corresponding to the $SU(N)$ sector of the $U(N)$ theory; those for the $U(1)$ sector, namely Z_{λ^0}, Z_{A^0} and Z_{g_0}, are given by omitting the terms in N and replacing g by g_0.)

Upon inserting Eq. (2.2) into Eq. (1.12) we obtain the one-loop contributions from S_B as

$$
\begin{aligned}
S_B^{(1)} = L\int d^4x\Big(& iC^{\mu\nu}\Big[\Big\{\frac{1}{2}(3+5\alpha)N + 2\Big\}g^3 d^{abc}\partial_\mu A_\nu^a\bar{\lambda}^b\bar{\lambda}^c + [3(\alpha-1)N+4]g^2 g_0 d^{ab0}\partial_\mu A_\nu^a\bar{\lambda}^b\bar{\lambda}^0\\
&+ 2[(3+\alpha)Ng^2 + g_0^2]\frac{g^2}{g_0}d^{0bc}\partial_\mu A_\nu^0\bar{\lambda}^b\bar{\lambda}^c + 2g_0^2 d^{000}\partial_\mu A_\nu^0\bar{\lambda}^0\bar{\lambda}^0\Big] - \Big[\frac{3}{2}(1+\alpha)N + 1\Big]id^{abe}f^{cde}g^4 C^{\mu\nu}A_\mu^c A_\nu^d\bar{\lambda}^a\bar{\lambda}^b\\
&- 2i(\alpha N + 1)d^{0be}f^{cde}g^3 g_0 C^{\mu\nu}A_\mu^c A_\nu^d\bar{\lambda}^0\bar{\lambda}^b - g^2|C|^2\Big[\frac{1}{4}[(3+2\alpha)N+1]g^2 d^{abe}d^{cde}(\bar{\lambda}^a\bar{\lambda}^b)(\bar{\lambda}^c\bar{\lambda}^d)\\
&+ \Big[(3+\alpha)\frac{g^4}{g_0^2} + \frac{g^2}{2N}\Big](\bar{\lambda}^a\bar{\lambda}^a)(\bar{\lambda}^b\bar{\lambda}^b) - \frac{Z_1^{(1)}}{N}g_0^2(\bar{\lambda}^a\bar{\lambda}^a)(\bar{\lambda}^0\bar{\lambda}^0)\Big] - \frac{1}{2}iZ_C^{(1)}C^{\mu\nu}d^{ABC}e^{ABC}F_{\mu\nu}^A\bar{\lambda}^B\bar{\lambda}^C\\
&+ Z_{|C|^2}^{(1)}\Big[\frac{1}{8}g^2|C|^2 d^{abe}d^{cde}(\bar{\lambda}^a\bar{\lambda}^b)(\bar{\lambda}^c\bar{\lambda}^d) + \frac{1}{4N}\frac{g^4}{g_0^2}|C|^2(\bar{\lambda}^a\bar{\lambda}^a)(\bar{\lambda}^b\bar{\lambda}^b)\Big]\\
&+ \Big\{\sqrt{2}C^{\mu\nu}[-((3+\alpha)Ng^2 + Z_2^{(1)} + 2\alpha\hat{C}_2)g\partial_\mu\bar{\phi}\bar{\lambda}^a R^a\bar{\sigma}_\nu\psi - 2\alpha\hat{C}_2 g_0\partial_\mu\bar{\phi}\bar{\lambda}^0 R^0\bar{\sigma}_\nu\psi - Z_2^{(1)}g\bar{\phi}\bar{\lambda}^a R^a\bar{\sigma}_\nu\partial_\mu\psi]\\
&+ i\sqrt{2}C^{\mu\nu}[g^2 A_\mu^b\bar{\phi}\bar{\lambda}^b[-Z_2^{(1)}R^a R^b + \Big(\frac{1}{2}Ng^2(9+3\alpha) + Z_2^{(1)}\Big)R^b R^a + 2\alpha\hat{C}_2]\bar{\sigma}_\nu\psi\\
&+ gg_0\Big(\frac{1}{2}Ng^2(3+\alpha) + 2\alpha\hat{C}_2\Big)A_\mu^b\bar{\phi}\bar{\lambda}^0 R^0 R^b\bar{\sigma}_\nu\psi + gg_0((3+\alpha)Ng^2 + 2\alpha\hat{C}_2)A_\mu^0\bar{\phi}\bar{\lambda}^a R^a R^0\bar{\sigma}_\nu\psi\\
&+ 2g_0^2\alpha\hat{C}_2 A_\mu^0\bar{\phi}\bar{\lambda}^0(R^0)^2\bar{\sigma}_\nu\psi] + iC^{\mu\nu}\bar{\phi}[(2(1-\alpha)\hat{C}_2 - (3+\alpha)Ng^2 - 2Z_2^{(1)})g\partial_\mu A_\nu^a R^a + 2(1-\alpha)\hat{C}_2 g_0\partial_\mu A_\nu^0 R^0\\
&+ ((\alpha-1)\hat{C}_2 + (3+\alpha)Ng^2 + Z_2^{(1)})g^2 f^{abc}A_\mu^a A_\nu^b R^c]F + \frac{1}{8}|C|^2[((1-\alpha)\hat{C}_2 - (6+2\alpha)Ng^2)g^2 d^{Abc}\bar{\phi}R^A\bar{\lambda}^b\bar{\lambda}^c\\
&+ 2((1-\alpha)\hat{C}_2 - (3+\alpha)Ng^2)g^2 d^{a0c}\bar{\phi}R^a\bar{\lambda}^0\bar{\lambda}^c + (1-\alpha)\hat{C}_2 d^{000}\bar{\phi}R^0\bar{\lambda}^0\bar{\lambda}^0]F\\
&+ Z_C^{(1)}C^{\mu\nu}[\sqrt{2}D_\mu\bar{\phi}\,\bar{\hat{\lambda}}\,\bar{\sigma}_\nu\psi + i\bar{\phi}\hat{F}_{\mu\nu}F] + \frac{1}{8}Z_{|C|^2}^{(1)}|C|^2 d^{ABC}\bar{\phi}R^A\bar{\hat{\lambda}}^B\bar{\hat{\lambda}}^C F\\
&+ (\phi\to\tilde{\phi}, \psi\to\tilde{\psi}, F\to\tilde{F}, R^A\to -(R^A)^*, C^{\mu\nu}\to -C^{\mu\nu})\Big\}\Big).
\end{aligned}
\tag{2.5}
$$

The results $\Gamma_{i1\mathrm{PI}}^{(1)\mathrm{pole}}$, $i = 1\ldots 8$ for the one-loop divergences from the 1PI graphs in Figs. 1–8 respectively are given in Appendix A. It is clear that they cannot be cancelled by Eq. (2.5), in particular since they contain contributions involving $\bar{\sigma}^{\mu\nu}$ which do not appear in Eq. (2.5). As we showed in Ref. [11], this can be remedied by field redefinitions, or, to put it another way, additional nonlinear field renormalizations. We find that a field redefinition

$$\delta\lambda^A = -\frac{1}{2}NLg^2 C^{\mu\nu}e^{BAC}d^{ABC}c^A c^B d^C\sigma_\mu\bar{\lambda}^C A_\nu^B, \tag{2.6}$$

where $c^A = 1 - \delta^{A0}$, $d^A = 1 + \delta^{A0}$, results in a change in the action

$$\delta S_\lambda = NLg^2 \int d^4x\Big[-\frac{1}{4} iC^{\mu\nu}(d^{abc} g \partial_\mu A^a_\nu \bar{\lambda}^b \bar{\lambda}^c - d^{abe} f^{cde} g^2 A^c_\mu A^d_\nu \bar{\lambda}^a \bar{\lambda}^b) + i d^{abc} g C^\mu{}_\nu A^a_\mu \bar{\lambda}^b \bar{\sigma}^{\nu\rho} \partial_\rho \bar{\lambda}^c$$
$$- \frac{1}{2} i d^{cde} f^{abe} g^2 C^{\mu\rho} A^c_\mu A^d_\nu \bar{\lambda}^a \bar{\sigma}^{\nu\rho} \bar{\lambda}^b + iC^{\rho\sigma} d^{a0c} g_0 A^a_\sigma \bar{\lambda}^0 [\delta^\mu{}_\rho + 2\bar{\sigma}^\mu{}_\rho]\partial_\mu \bar{\lambda}^c + iC^{\mu\nu} f^{abc} d^{cd0} g g_0 A^b_\mu A^d_\nu \bar{\lambda}^a \bar{\lambda}^0$$
$$- \Big\{\frac{1}{2} i\sqrt{2} g^2 C^{\mu\nu} d^{abc} A^b_\mu \bar{\phi} R^c \bar{\lambda}^a \bar{\sigma}_\nu \psi + i\sqrt{2} g g_0 C^{\mu\nu} d^{0bc} A^b_\mu \bar{\phi} R^c \bar{\lambda}^0 \bar{\sigma}_\nu \psi$$
$$+ (\phi \to \tilde{\phi}, \psi \to \tilde{\psi}, F \to \tilde{F}, R^A \to -(R^A)^*, C^{\mu\nu} \to -C^{\mu\nu})\Big\}\Big], \tag{2.7}$$

which miraculously casts all the C-dependent terms apart from those linear in F, $\tilde{F}$ into the correct form. Then finally redefinitions of $\bar{F}$, $\bar{\tilde{F}}$ can be used to deal with the terms linear in F, $\tilde{F}$. Explicitly, we need

$$\delta\bar{F} = L\Big\{\Big[(5Ng^2 + 2(1+\alpha)\hat{C}_2) g \partial_\mu A^a_\nu - \Big(\frac{11}{4} Ng^2 + (1+\alpha)\hat{C}_2\Big) g^2 f^{abc} A^b_\mu A^c_\nu\Big] iC^{\mu\nu}\bar{\phi} R^a + 2(\alpha+1)\hat{C}_2 g_0 \partial_\mu A^0_\nu iC^{\mu\nu} \bar{\phi} R^0$$
$$+ \frac{1}{8}|C^2|[(-37Ng^2 + (63+\alpha)\hat{C}_2) g^2 d^{abc} \bar{\phi} R^c \bar{\lambda}^a \bar{\lambda}^b + (-32Ng^2 + (31+\alpha)\hat{C}_2) g g_0 d^{0bc} \bar{\phi} R^c \bar{\lambda}^0 \bar{\lambda}^b$$
$$+ (31+\alpha)\hat{C}_2 g_0^2 d^{000} \bar{\phi} R^0 \bar{\lambda}^0 \bar{\lambda}^0 + (6Ng^2 + (\alpha-1)\hat{C}_2) g^2 d^{ab0} \bar{\phi} R^0 \bar{\lambda}^a \bar{\lambda}^b]\Big\} \tag{2.8}$$

(with a similar redefinition of $\bar{\tilde{F}}$) which produce a change in the action

$$\delta S_F = \int d^4x (\delta\bar{F} F + \delta\bar{\tilde{F}}\tilde{F}). \tag{2.9}$$

We now find (writing for instance $Z_C^{(n)}$ for the n-loop contribution to Z_C) that with

$$Z_C^{(1)} = Z_{|C|^2}^{(1)} = 0, \qquad Z_1^{(1)} = -3Ng^2, \qquad Z_2^{(1)} = -Ng^2, \tag{2.10}$$

we have

$$\Gamma^{(1)\mathrm{pole}\prime} = \sum_{i=1}^{8} \Gamma^{(1)\mathrm{pole}}_{i1\mathrm{PI}} + \delta S_\lambda + \delta S_F + S_B^{(1)} = 0, \tag{2.11}$$

i.e. $\Gamma^{(1)\prime}$ is finite.

This demonstrates that our theory is renormalizable and that the $\mathcal{N} = \frac{1}{2}$ supersymmetry is preserved. However we find that to obtain a renormalizable lagrangian it is vital (since $Z_1^{(1)}, Z_2^{(1)} \neq 0$) to include the terms involving γ_1, γ_2 in Eq. (1.12), which were not in the original formulation of the theory [4] though they are independently $\mathcal{N} = \frac{1}{2}$ supersymmetric. This is not unexpected since in general any terms which are not forbidden by a symmetry will be generated under renormalization. It is therefore all the more remarkable that we do not need to renormalize the nonanticommutativity parameter C and that the other $\bar{\lambda}^4$ terms (which are also separately $\mathcal{N} = \frac{1}{2}$ supersymmetric) do not require any counterterms. On the other hand our renormalized lagrangian is no longer of the form derived from nonanticommutative superspace. Of course this was also found in the case of the $\mathcal{N} = \frac{1}{2}$ Wess-Zumino model [7].

We note here that the requirement to make a divergent redefinition of $\bar{F}$ is not as surprising as it may first appear (if calculating in components with a conventional covariant gauge). In fact, if one renormalizes the ordinary $\mathcal{N} = 1$ theory in its uneliminated component form, i.e. before eliminating the auxiliary fields, one is compelled to make a similar nonlinear renormalization of F to render the theory finite. This has not to our knowledge previously been discussed, and we give the details in a forthcoming publication [14].

III. CONCLUSIONS

We have studied the renormalizability of a general $\mathcal{N} = \frac{1}{2}$ supersymmetric theory coupled to chiral matter. The nonrenormalizability of the standard $U(N)$ version was apparent from the outset, and it appeared impossible to define a general $SU(N)$ invariant $\mathcal{N} = \frac{1}{2}$ supersymmetric theory; however we were able to define an $SU(N) \otimes U(1)$ invariant action which still possessed $\mathcal{N} = \frac{1}{2}$ supersymmetry, which as we showed was preserved under renormalization. Moreover we find that the nonanticommutativity parameter C is unrenormalized (at least at one loop).

We have restored gauge invariance by a somewhat unconventional expedient which works rather miraculously. One could speculate to what extent the $\mathcal{N} = \frac{1}{2}$ supersymmetry and the identities Eq. (1.9) were required to make this trick work. If one treats the action (1.1) as primordial, ignoring its derivation from nonanticommuting superspace, the identities Eq. (1.9) can be regarded as a consequence of the self-duality of $C^{\mu\nu}$ (with $C^{\alpha\beta}$ now *defined* by

Eq. (1.9a)). It would be interesting to examine a theory of the same form but in which $C^{\mu\nu}$ was replaced by a general antisymmetric tensor. Moreover, suppose one considered a theory with an action based on Eq. (1.1) but including all the hermitian conjugate terms which are missing. The only new diagrams would simply be the "hermitian conjugates" of those in Figs. 1–8. Equation (2.6) would now need to be supplemented by its hermitian conjugate. However, the variation of the action would now include additional unwanted non-gauge-invariant terms since it is now not only the gaugino kinetic term which varies. This raises the possibility of a theory (albeit nonrenormalizable) with ineradicable non-gauge-invariant divergences.

An interesting feature of our results is the redefinition (or nonlinear renormalization) of the gaugino field. As we have mentioned, the attendant nonlinear redefinition of the auxiliary field F has its counterpart even in the $\mathcal{N}=1$ theory, so that nonlinear field redefinitions may be an unavoidable consequence of working in the uneliminated component formalism with conventional gauge-fixing; as we mentioned, no such field redefinition was required in the $\mathcal{N}=\frac{1}{2}$ superfield calculation of Ref. [12].

Obviously it would be valuable to continue the renormalization programme beyond one loop, and also to include a superpotential. Of course we would expect that γ_{1B} and γ_{2B} in Eq. (2.1) would get contributions from all loop orders, but it would be interesting to see whether any additional counterterms need to be introduced and furthermore whether the relation $Z_C = Z_{|C|^2} = 1$ persists to higher orders. (The apparent violation of this result in the case of the ungauged Wess-Zumino model at one and two loops [7] occurs in a separately $\mathcal{N}=\frac{1}{2}$ invariant term, F^3, for which we would assign a separate coupling in a similar fashion to γ_{1B} and γ_{2B}.)

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APPENDIX A: RESULTS FOR ONE-LOOP DIAGRAMS

The divergent contributions to the effective action from the graphs in Fig. 1 are of the form:

$$ig^2 L d^{ABC} e^{ABC} C^{\mu\nu}\Big[\partial_\mu A^A_\rho \bar{\lambda}^B (T_1^{ABC}\delta_\nu{}^\rho + \tilde{A}_1^{ABC}\bar{\sigma}_\nu{}^\rho)\bar{\lambda}^C + A^A_\rho \bar{\lambda}^B(\tilde{T}_1^{ABC}\delta_\nu{}^\rho + A_1^{ABC}\bar{\sigma}_\nu{}^\rho)\partial_\mu\bar{\lambda}^C\Big], \tag{A1}$$

where the contributions to T_1, $\tilde{T}_1$, A_1, $\tilde{A}_1$ from the individual graphs are given in Table I.

In Table I, $g^{a0b} = g^{ab0} = g^{0ab} = g^{000} = g_0$ and $g^{abc} = g$.

We note here that Figs. 1(f)–1(h) correspond to both ϕ, ψ and $\tilde{\phi}$, $\tilde{\psi}$ loops, which contribute identically due to the change in sign $C^{\mu\nu} \to -C^{\mu\nu}$ between the ϕ, ψ and $\tilde{\phi}$, $\tilde{\psi}$ interactions in the lagrangian. Potential contributions of the form $gLf^{ABC}C^{\mu\nu}\partial_\mu A^A_\rho\bar{\lambda}^B\bar{\sigma}_\nu{}^\rho\bar{\lambda}^C$, $gLf^{ABC}C^{\mu\nu}A^A_\rho\bar{\lambda}^B\delta_\nu{}^\rho\partial_\mu\bar{\lambda}^C$ cancel between ϕ, ψ and $\tilde{\phi}$, $\tilde{\psi}$ loops.

The divergent contributions to the effective action from the graphs in Fig. 1 are given by

$$\begin{aligned}\Gamma^{(1)\mathrm{pole}}_{11\mathrm{PI}} &= ig^2 L C^{\mu\nu} d^{ABC} e^{ABC}\bigg[\partial_\mu A^A_\rho \bar{\lambda}^B\bigg(\bigg[-(1+2\alpha)Nc^A d^B c^C + \frac{1}{2}(5+\alpha)Nc^A - (3+\alpha)Nd^A c^B c^C - 2\frac{g^{ABC}}{e^{ABC}}\bigg]\delta_\nu{}^\rho \\ &\quad + \frac{2}{3}N[c^A d^B c^C - c^A c^B d^C]\bar{\sigma}_\nu{}^\rho\bigg)\bar{\lambda}^C + A^A_\rho\bar{\lambda}^B\bigg(-\frac{1}{2}Nc^A d^B c^C\delta_\nu{}^\rho + \frac{1}{3}N[c^A d^B c^C - 4c^A c^B d^C]\bar{\sigma}_\nu{}^\rho\bigg)\partial_\mu\bar{\lambda}^C\bigg] \\ &= iLC^{\mu\nu}\bigg[-\bigg\{\frac{5}{4}(1+2\alpha)N + 2\bigg\}g^3 d^{abc}\partial_\mu A^a_\nu\bar{\lambda}^b\bar{\lambda}^c + [3(1-\alpha)N - 4]g^2 g_0 d^{ab0}\partial_\mu A^a_\nu\bar{\lambda}^b\bar{\lambda}^0 \\ &\quad - 2[(3+\alpha)Ng^2 + g_0^2]\frac{g^2}{g_0}d^{0bc}\partial_\mu A^0_\nu\bar{\lambda}^b\bar{\lambda}^c - 2g^2 g_0 d^{000}\partial_\mu A^0_\nu\bar{\lambda}^0\bar{\lambda}^0 - Ng^2 g_0 d^{a0c}A^a_\nu\bar{\lambda}^0\partial_\mu\bar{\lambda}^c \\ &\quad - Ng^3 d^{abc}A^a_\rho\bar{\lambda}^b\bar{\sigma}_\nu{}^\rho\partial_\mu\bar{\lambda}^c - 2Ng^2 g_0 d^{a0c}A^a_\rho\bar{\lambda}^0\bar{\sigma}_\nu{}^\rho\partial_\mu\bar{\lambda}^c].\end{aligned} \tag{A2}$$

The divergent contributions to the effective action from the graphs in Fig. 2 are of the form:

$$ig^3 L e^{EAB}[d^{ABE} f^{CDE} C^{\mu\nu} T_2^{ABCD} A^C_\mu A^D_\nu \bar{\lambda}^A\bar{\lambda}^B + d^{CDE} f^{ABE} C^{\mu\rho} A_2^{ABCD} A^C_\mu A^D_\nu \bar{\lambda}^A \bar{\sigma}^\nu{}_\rho \bar{\lambda}^B] \tag{A3}$$

where the contributions to T_2, A_2 from the individual graphs are given in Table II.

The contributions from Figs. 2(m)–2(o)are zero. The graphs in Fig. 2 add to

TABLE I. Contributions to T_1, $\tilde{T}_1$, A_1, $\tilde{A}_1$ from Fig. 1

Graph	T_1	$\tilde{T}_1$	$\tilde{A}_1$	A_1
1a	$-(3+\alpha)Nd^Ac^Bc^C$	0	0	0
1b	$-Nc^Ad^Bc^C$	0	$-\frac{2}{3}Nc^Ac^Bd^C$	$-\frac{4}{3}Nc^Ac^Bd^C$
1c	$-2\alpha Nc^Ad^Bc^C$	$\frac{1}{2}(2-\alpha)Nc^Ad^Bc^C$	$\frac{2}{3}Nc^Ad^Bc^C$	$-\frac{1}{3}(2+3\alpha)Nc^Ad^Bc^C$
1d	$\frac{1}{2}(5+\alpha)Nc^A$	0	0	0
1e	0	$-\frac{1}{2}(3-\alpha)Nc^Ad^Bc^C$	0	$(1+\alpha)Nc^Ad^Bc^C$
1f	$-g^{ABC}/e^{ABC}$	0	0	$-\frac{4}{3}g^{ABC}/e^{ABC}$
1g	$-\frac{1}{2}g^{ABC}/e^{ABC}$	0	0	$-\frac{2}{3}g^{ABC}/e^{ABC}$
1h	$-\frac{1}{2}g^{ABC}/e^{ABC}$	0	0	$2g^{ABC}/e^{ABC}$

$$\begin{aligned}\Gamma^{(1)\text{pole}}_{21\text{PI}} &= \frac{1}{4}ig^3L[2(1+2\alpha)Nd^Ac^B + 2(3+\alpha)Nc^Ac^B - 3N - 2N\delta^{A0} + 4]e^{EAB}d^{ABE}f^{CDE}C^{\mu\nu}A^C_\mu A^D_\nu\bar{\lambda}^A\bar{\lambda}^B \\ &\quad + \frac{1}{2}ig^3NLd^Ac^Bc^Cc^De^{EAB}d^{CDE}f^{ABE}C^{\mu\rho}A^C_\mu A^D_\nu\bar{\lambda}^A\bar{\sigma}^\nu{}_\rho\bar{\lambda}^B \\ &= \left[\frac{1}{4}(5+6\alpha)N+1\right]iLd^{abe}f^{cde}g^4C^{\mu\nu}A^c_\mu A^d_\nu\bar{\lambda}^a\bar{\lambda}^b + \frac{1}{2}iNLd^{cde}f^{abe}g^4C^{\mu\rho}A^c_\mu A^d_\nu\bar{\lambda}^a\bar{\sigma}^\nu{}_\rho\bar{\lambda}^b \\ &\quad - iL[(1-2\alpha)N-2]d^{0be}f^{cde}g^3g_0C^{\mu\nu}A^c_\mu A^d_\nu\bar{\lambda}^0\bar{\lambda}^b. \end{aligned} \tag{A4}$$

The results for Fig. 3 are of the form:

$$g^2L|C|^2[X_1^{abcd}(\bar{\lambda}^a\bar{\lambda}^b)(\bar{\lambda}^c\bar{\lambda}^d) + X_2(\bar{\lambda}^a\bar{\lambda}^a)(\bar{\lambda}^b\bar{\lambda}^b) + X_3(\bar{\lambda}^a\bar{\lambda}^a)(\bar{\lambda}^0\bar{\lambda}^0)] \tag{A5}$$

where the contributions to X_{1-3} are given in Table III.

In Table III,

$$d^{abcd} = \text{Tr}\,[F^aF^bD^cD^d], \qquad \tilde{d}^{abcd} = \text{Tr}\,[F^aD^cF^bD^d], \tag{A6}$$

where the matrices F^a and D^a are defined in Appendix B. These results add to

TABLE II. Contributions from Fig. 2

Graph	T_2	A_2
2a	$\frac{1}{2}Nd^Ac^B$	$\frac{1}{3}Nd^Ac^Bc^Cc^D$
2b	$-\frac{1}{2}(3-\alpha)N\delta^{A0}$	
2c	$\frac{1}{2}(3+\alpha)Nc^Ac^B$	0
2d	$\frac{1}{2}(2-\alpha)N\delta^{A0}$	
2e	$-\frac{1}{2}\alpha Nd^Ac^B$	$\frac{1}{6}(4+3\alpha)Nd^Ac^Bc^Cc^D$
2f	$\frac{3}{4}\alpha Nd^Ac^B$	$-\frac{1}{2}(2+\alpha)Nd^Ac^Bc^Cc^D$
2g	$\frac{3}{4}\alpha Nd^Ac^B$	$\frac{1}{2}Nd^Ac^Bc^Cc^D$
2h	$-\frac{3}{4}(1+\alpha)N$	0
2i	$\frac{3}{4}\alpha N$	0
2j	$\frac{1}{2}$	$\frac{1}{3}$
2k	0	$\frac{2}{3}$
2l	$\frac{1}{2}$	-1

$$\begin{aligned}\Gamma^{(1)\text{pole}}_{31\text{PI}} &= g^2L|C|^2\left[\frac{1}{4}[(3+2\alpha)N+1]g^2d^{abe}d^{cde}(\bar{\lambda}^a\bar{\lambda}^b)\right. \\ &\quad \times(\bar{\lambda}^c\bar{\lambda}^d) + \frac{1}{2N}\left[2(3+\alpha)N\frac{g^4}{g_0^2} + g^2\right] \\ &\quad \left.\times(\bar{\lambda}^a\bar{\lambda}^a)(\bar{\lambda}^b\bar{\lambda}^b) + 3g_0^2(\bar{\lambda}^a\bar{\lambda}^a)(\bar{\lambda}^0\bar{\lambda}^0)\right]. \end{aligned} \tag{A7}$$

In obtaining these results we have made frequent use of the Fierz identity

$$(\bar{\lambda}^a\bar{\lambda}^b)(\bar{\lambda}^c\bar{\lambda}^d) + (\bar{\lambda}^a\bar{\lambda}^c)(\bar{\lambda}^b\bar{\lambda}^d) + (\bar{\lambda}^a\bar{\lambda}^d)(\bar{\lambda}^b\bar{\lambda}^c) = 0 \tag{A8}$$

The contributions from the graphs shown in Fig. 4 are of the form

$$\sqrt{2}g_ALC^{\mu\nu}\partial_\mu\bar{\phi}\bar{\lambda}^AX^A\bar{\sigma}_\nu\psi + \sqrt{2}g_ALC^{\mu\nu}\bar{\phi}\bar{\lambda}^AY^A\bar{\sigma}_\nu\partial_\mu\psi \tag{A9}$$

where $g_a \equiv g$ and X^A and Y^A are as given in Table IV. (Here and elsewhere there are analogous diagrams with $\tilde{\phi}$, $\tilde{\psi}$, $\tilde{F}$ external legs which we do not show explicitly; their contributions may easily be read off using $\phi \to \tilde{\phi}$, $\psi \to \tilde{\psi}$, $F \to \tilde{F}$, $R^A \to -(R^A)^*$, $C^{\mu\nu} \to -C^{\mu\nu}$.)

These graphs add to

TABLE III. Contributions from Fig. 3

Graph	X_1^{abcd}	X_2	X_3
3a	$\frac{1}{4}(3+\alpha)Ng^2d^{abe}d^{cde}+2g^2d^{abcd}+(1-\alpha)g^2d^{adcb}-\frac{4}{N}\frac{g^4}{g_0^2}f^{eac}f^{ebd}$	$(3+\alpha)\frac{g^4}{g_0^2}$	0
3b	$\frac{1}{2}\alpha Ng^2d^{abe}d^{cde}$	0	$-2\alpha g_0^2$
3c	$-\frac{1}{4}(1+\alpha)Ng^2d^{abe}d^{cde}$	0	$(1+\alpha)g_0^2$
3d	$g^2[-2d^{abcd}+(\alpha-1)d^{adcb}+\frac{4}{N}\frac{g^2}{g_0^2}f^{eac}f^{ebd}]$	0	$(3+\alpha)g_0^2$
3e	$\frac{1}{3}g^2(\tilde{d}^{abcd}-\tilde{d}^{acdb})$	0	$-g_0^2$
3f	$\frac{1}{4}g^2d^{abe}d^{cde}$	$\frac{1}{2N}g^2$	0

$$\begin{aligned}\Gamma_{41\text{PI}}^{(1)\text{pole}} &= \sqrt{2}g_ALC^{\mu\nu}\partial_\mu\bar{\phi}\bar{\lambda}^A[2\alpha\hat{C}_2R^A+(2+\alpha)Ng^2c^AR^A]\bar{\sigma}_\nu\psi-\sqrt{2}Ng_Ag^2c^AC^{\mu\nu}\bar{\phi}\bar{\lambda}^AR^A\bar{\sigma}_\nu\partial_\mu\psi\\ &= L\{[2\alpha\hat{C}_2+(2+\alpha)Ng^2]\sqrt{2}gC^{\mu\nu}\partial_\mu\bar{\phi}\bar{\lambda}^aR^a\bar{\sigma}_\nu\psi+2\alpha\hat{C}_2\sqrt{2}g_0C^{\mu\nu}\partial_\mu\bar{\phi}\bar{\lambda}^0R^0\bar{\sigma}_\nu\psi-N\sqrt{2}g^3C^{\mu\nu}\bar{\phi}\bar{\lambda}^aR^a\bar{\sigma}_\nu\partial_\mu\psi\}.\end{aligned} \tag{A10}$$

The contributions from the graphs shown in Fig. 5 are of the form

$$\sqrt{2}ig_Ag_BLC^{\mu\nu}A_\mu^B\bar{\phi}\bar{\lambda}^AZ^{AB}\bar{\sigma}_\nu\psi \tag{A11}$$

where in the case of Figs. 5(a)–5(v), Z^{AB} contains the contributions shown in Table V.

The contributions from Table Va add to

$$i\sqrt{2}g_Ag_BLC^{\mu\nu}A_\mu^B\bar{\phi}\bar{\lambda}^A\Big[4\hat{C}_2R^AR^B+(8-2\alpha)\hat{C}_2R^BR^A+Ng^2(-4R^AR^Bc^A+2R^AR^Bc^B-(2+\alpha)R^BR^Ac^A-\frac{1}{2}(7+\alpha)R^BR^Ac^B)\Big]\bar{\sigma}_\nu\psi. \tag{A12}$$

In the case of Figs. 5(w)–5(z), 5(aa), 5(bb), and 5(cc), the contributions to Z^{ab} are shown in Table Vb.

The contributions to Z^{0b} from Figs. 5(w)–5(z), 5(aa), 5(bb), and 5(cc) are shown in Table V c.

The contributions to Z^{a0} and Z^{00} from Figs. 5(w)–5(z), 5(aa), 5(bb), and 5(cc) are shown in Table Vd (those not shown explicitly are zero). Adding the results from Table Va in Eq. (A12) to those from Tables Vb–d, we obtain

$$\begin{aligned}\Gamma_{51\text{PI}}^{(1)\text{pole}} = i\sqrt{2}NLC^{\mu\nu}\Big[&g^4A_\mu^b\bar{\phi}\bar{\lambda}^a\Big[\frac{1}{2}d^{abc}R^c-R^aR^b-\frac{1}{2}(7+3\alpha)R^bR^a\Big]\bar{\sigma}_\nu\psi\\ &+g^3g_0A_\mu^b\bar{\phi}\bar{\lambda}^0\Big[d^{0bc}R^c-\frac{1}{2}(3+\alpha)R^0R^b\Big]\bar{\sigma}_\nu\psi-(3+\alpha)g^3g_0A_\mu^0\bar{\phi}\bar{\lambda}^aR^aR^0\bar{\sigma}_\nu\psi\\ &-2\frac{\alpha}{N^2}\hat{C}_2A_\mu^A\bar{\phi}\bar{\lambda}^Bg_Ag_BR^BR^A\bar{\sigma}_\nu\psi\Big].\end{aligned} \tag{A13}$$

TABLE IV. Contributions to X^A and Y^A from Fig. 4

Graph	X^A	Y^A
4a	$\frac{3}{2}Ng^2c^AR^A$	$-\alpha Nc^AR^A$
4b	$\alpha Ng^2c^AR^A$	αNc^AR^A
4c	$\alpha Ng^2c^AR^A$	0
4d	$-2[\hat{C}_2-\frac{1}{2}Ng^2c^A]R^A$	$2[\hat{C}_2-\frac{1}{2}Ng^2c^A]R^A$
4e	$-2[\hat{C}_2-\frac{1}{2}Ng^2c^A]R^A$	0
4f	$-(1-2\alpha)[\hat{C}_2-\frac{1}{2}Ng^2c^A]R^A$	0
4g	$2[2\hat{C}_2-\frac{1}{2}Ng^2c^A]R^A$	0
4h	$2[2\hat{C}_2-\frac{1}{2}Ng^2c^A]R^A$	$-2[2\hat{C}_2-\frac{1}{2}Ng^2c^A]R^A$
4i	0	$2[\hat{C}_2-\frac{1}{2}Ng^2c^A]R^A$
4j	$-3\hat{C}_2R^A$	0

The contributions from Fig. 6 are of the form

$$iLC^{\mu\nu}(g_A\partial_\mu A_\nu^A\bar{\phi}XR^AF+g_AA_\nu^A\partial_\mu\bar{\phi}YR^AF) \tag{A14}$$

where X and Y are given in Table VI.

The contributions in Table VI add to

$$\begin{aligned}\Gamma_{61\text{PI}}^{(1)\text{pole}} &= -iLC^{\mu\nu}\bar{\phi}g_A\partial_\mu A_\nu^A[4\hat{C}_2+(4-\alpha)Ng^2c^A]R^AF\\ &= -iLC^{\mu\nu}\bar{\phi}\{g[4\hat{C}_2+(4-\alpha)Ng^2]\partial_\mu A_\nu^aR^a\\ &\quad+4g_0\hat{C}_2\partial_\mu A_\nu^0R^0\}F.\end{aligned} \tag{A15}$$

The contributions from Fig. 7 are of the form

TABLE V. (a) Contributions to Z^{AB} from Figs. 5(a)–5(v). (b) Contributions to Z^{AB} from Fig. 5(w)–5(z), 5(aa), 5(bb), and 5(cc). (c) Contributions to Z^{0b} from Fig. 5(w)–5(z), 5(aa), 5(bb), and 5(cc). (d) Contributions to Z^{0b} from Fig. 5(w)–5(z), 5(aa), 5(bb), and 5(cc).

Graph	$\hat{C}_2 R^A R^B$	$\hat{C}_2 R^B R^A$	$Ng^2 R^A R^B$	$Ng^2 R^B R^A$	$g^2 f^{ACE} f^{BDE} R^C R^D$
5a	0	2	$-c^A$	$-c^B$	$2c^A c^B$
5b	-2	0	$c^A + c^B$	0	$-2c^A c^B$
5c	0	0	$2c^B$	0	$-4c^A c^B$
5d	4	0	$-2(c^A + c^B)$	0	$4c^A c^B$
5e	0	2	0	$-c^A$	0
5f	0	0	$\frac{1}{2}(1-\alpha)c^B$	0	$(\alpha - 1)c^A c^B$
5g	2	0	$-(c^A + c^B)$	0	$2c^A c^B$
5h	0	$-\alpha$	0	$\frac{1}{2}\alpha c^B$	0
5i	0	0	0	$-\frac{3}{4}\alpha c^B$	0
5j	0	$3+\alpha$	0	$-\frac{1}{4}(3+\alpha)c^B$	0
5k	0	0	$-\frac{1}{4}\alpha c^B$	0	$\frac{1}{2}\alpha c^A c^B$
5l	0	$1-\alpha$	$\frac{1}{4}(\alpha - 1)c^A$	$\frac{1}{4}(\alpha-1)(c^A + c^B)$	$\frac{1}{2}(1-\alpha)c^A c^B$
5m	0	-2α	αc^A	αc^B	$-2\alpha c^A c^B$
5n	0	0	$-\alpha c^A$	0	$2\alpha c^A c^B$
5o	0	α	$-\frac{1}{2}\alpha c^A$	$-\frac{1}{2}\alpha c^B$	$\alpha c^A c^B$
5p	0	0	$-\frac{1}{4}(3+\alpha)c^A$	$-\frac{1}{4}(3+\alpha)c^A$	$\frac{1}{2}(3+\alpha)c^A c^B$
5q	0	0	0	$-\alpha c^A$	0
5r	0	0	$\frac{1}{2}\alpha c^A$	0	$-\alpha c^A c^B$
5s	0	0	$\frac{3}{2}(1+\alpha)c^B$	$-\frac{3}{2}(1+\alpha)c^B$	$-3(1+\alpha)c^A c^B$
5t	0	0	0	0	$-2\alpha c^A c^B$
5u	0	0	0	0	$2\alpha c^A c^B$
5v	0	0	$-\frac{3}{4}\alpha c^B$	$\frac{3}{4}\alpha c^B$	$\frac{3}{2}\alpha c^A c^B$

Graph	$Ng^2 R^a R^b$	$Ng^2 R^b R^a$	$\frac{1}{N}\Delta R^a R^b$	$\frac{1}{N}\Delta R^b R^a$	$g^2 \delta^{ab}$
5w	$-\frac{1}{2}(3+\alpha)$	0	$-(3+\alpha)$	$3+\alpha$	$\frac{1}{4}(3+\alpha)$
5x	0	-1	0	0	$\frac{1}{2}$
5y	0	0	0	-4	-1
5z	$\frac{1}{2}(2+\alpha)$	0	$2+\alpha$	$-(2+\alpha)$	$-\frac{1}{4}(2+\alpha)$
5aa	$-\frac{1}{2}\alpha$	0	α	$-\alpha$	$\frac{1}{4}\alpha$
5bb	0	$-\frac{1}{2}$	-1	-1	$-\frac{1}{4}$
5cc	$\frac{1}{2}\alpha$	0	$-\alpha$	α	$-\frac{1}{4}\alpha$

Graph	$Ng^2 R^b R^0$	$\frac{1}{N}\Delta R^0 R^b$
5w	$-3+\alpha$	0
5x	-2	0
5y	0	-4
5z	$2+\alpha$	0
5aa	$-\alpha$	0
5bb	-1	-2
5cc	α	0

Graph	$(a0)$	(00)
	$g^2 R^a R^0 + 2\frac{1}{N}\Delta R^a R^0$	$g^2 + \frac{1}{N^2}\Delta$
5y	-2	-4
5bb	-1	-2

TABLE VI. Contributions from Fig. 6

Graph	X	Y
6a	0	$3Ng^2c^A$
6b	0	$2[2\hat{C}_2 - Ng^2c^A]$
6c	$-[4\hat{C}_2 - Ng^2c^A]$	$-[4\hat{C}_2 - Ng^2c^A]$
6d	$-(5+\alpha)Ng^2c^A$	0
6e	$2\alpha Ng^2c^A$	$-2Ng^2c^A$

TABLE VII. Contributions from Fig. 7

Graph	Z
7a	$-\frac{3}{4}\alpha Ng^2$
7b	0
7c	0
7d	0
7e	$-\frac{1}{4}(2+\alpha)Ng^2$
7f	$2\hat{C}_2 - Ng^2$
7g	$-\frac{3}{2}\alpha Ng^2$
7h	$\frac{3}{2}(1+\alpha)Ng^2$
7i	$\frac{1}{4}(3+\alpha)Ng^2$
7j	$\frac{1}{2}\alpha Ng^2$
7k	$-\frac{3}{4}\alpha Ng^2$
7l	0

$$ig^2LC^{\mu\nu}A^a_\mu A^b_\nu\bar{\phi}Zf^{abc}R^cF \tag{A16}$$

where Z is given in Table VII.

The contributions in Table VII add to

$$\Gamma^{(1)\mathrm{pole}}_{71\mathrm{PI}} = ig^2LC^{\mu\nu}A^a_\mu A^b_\nu\bar{\phi}\left[2\hat{C}_2 + \frac{1}{4}(3-4\alpha)Ng^2\right] \times f^{abc}R^cF \tag{A17}$$

The contributions from Fig. 8 are of the form

$$Lg_Ag_B|C|^2\bar{\lambda}^A\bar{\lambda}^B\bar{\phi}Z^{AB}F \tag{A18}$$

where the contributions to Z^{AB} are given in Table VIII. The contributions in Table VIII add to

$$\begin{aligned}\Gamma^{(1)\mathrm{pole}}_{81\mathrm{PI}} = L|C|^2\bar{\phi}\Big\{&g^2\left[\frac{1}{8}(43+2\alpha)Ng^2 - 8\hat{C}_2\right]\bar{\lambda}^a\bar{\lambda}^bd^{abc}R^c\\ &+ gg_0\left[\frac{1}{4}(19+\alpha)Ng^2 - 8\hat{C}_2\right]\bar{\lambda}^0\bar{\lambda}^bd^{0bc}R^c\\ &+\frac{1}{4}\alpha Ng^4d^{0bc}R^0\bar{\lambda}^b\bar{\lambda}^c - 4g_0^2\hat{C}_2d^{000}\bar{\lambda}^0\bar{\lambda}^0R^0\Big\}F.\end{aligned} \tag{A19}$$

APPENDIX B: GROUP IDENTITIES FOR $SU(N)$

The basic commutation relations for $SU(N)$ are (for the fundamental representation):

$$[R^a, R^b] = if^{abc}R^c, \qquad \{R^a, R^b\} = d^{abc}R^c + \frac{1}{N}\delta^{ab}, \tag{B1}$$

where d^{abc} is totally symmetric. Defining matrices F^a, D^a by $(F^a)^{bc} = f^{bac}$, $(D^a)^{bc} = d^{bac}$, useful identities for $SU(N)$ are

$$\begin{aligned}\mathrm{Tr}\,[F^aF^b] &= -N\delta^{ab},\\ \mathrm{Tr}\,[D^aD^b] &= \frac{N^2-4}{N}\delta^{ab},\\ \mathrm{Tr}\,[F^aF^bD^c] &= -\frac{N}{2}d^{abc},\\ \mathrm{Tr}\,[F^aD^bD^c] &= \frac{N^2-4}{2N}f^{abc},\\ C_2(R) &= \frac{N^2-1}{2N},\\ \mathrm{Tr}\,[F^aD^bF^cD^d] &= \frac{N}{4}(d^{acx}d^{bdx} - d^{abx}d^{cdx} - d^{adx}d^{bcx}).\end{aligned} \tag{B2}$$

TABLE VIII. Contributions from Fig. 8

Graph	ab	$a0$	00
8a	0	0	0
8b	$-g^2\delta^{ab} - \frac{4}{N}\Delta R^aR^b$	$-\sqrt{\frac{N}{2}}[g^2 + \frac{4}{N^2}\Delta]R^a$	$-2g^2 - \frac{2}{N^2}\Delta$
8c	$\frac{1}{2}g^2\delta^{ab} + \frac{1}{N}\Delta R^aR^b$	$\frac{1}{4}(\frac{2}{N})^{3/2}\Delta R^a$	$\frac{1}{2}g^2 + \frac{1}{2N^2}\Delta$
8d	$-\alpha g^2Nd^{abc}R^c$	$-\alpha g^2\sqrt{2N}R^a$	0
8e	$(1+\alpha)g^2Nd^{abc}R^c$	$(1+\alpha)g^2\sqrt{2N}R^a$	0
8f	$-\frac{1}{2}\alpha g^2Nd^{abc}R^c$	$-\frac{1}{2}\alpha g^2\sqrt{2N}R^a$	0
8g	0	0	0
8h	$\frac{1}{2}\alpha g^2Nd^{abc}R^c$	$\frac{1}{2}\alpha g^2\sqrt{2N}R^a$	0
8i	$\frac{1}{4}(3+\alpha)g^2[\frac{1}{2}Nd^{abc}R^c + \delta^{ab}]$	0	0
8j	$\frac{1}{8}\alpha g^2Nd^{abc}R^c$	$\frac{1}{8}\alpha g^2\sqrt{2N}R^a$	0
8k	$-\frac{1}{4}g^2\delta^{ab} - \frac{1}{N}\Delta R^aR^b$	$-\frac{1}{4}\sqrt{\frac{N}{2}}[g^2 + \frac{4}{N^2}\Delta]R^a$	$-\frac{1}{2}g^2 - \frac{1}{2N^2}\Delta$

$$d^{acd}R^bR^cR^d = \frac{N^2-4}{2N}R^bR^a, \qquad d^{ace}f^{bde}R^cR^d = i\left[-\frac{1}{2}NR^aR^b + \frac{1}{N}[R^a, R^b] + \frac{1}{4}\delta^{ab}\right],$$
$$d^{ace}f^{bde}R^dR^c = i\left[\frac{1}{2}NR^bR^a + \frac{1}{N}[R^a, R^b] - \frac{1}{4}\delta^{ab}\right], \qquad d^{acd}R^cR^bR^d = -\frac{1}{N}\{R^a, R^b\} + \frac{1}{4}\delta^{ab}. \tag{B3}$$

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